Test the Practicability of Markowitz Model for Modern Investors

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Abstract: As the basis of modern investment theory, the Markowitz model can still play a vital role in the investment scheme of modern investors. When the influencing factors of modern stock market are becoming more and more complex, by clarifying the mathematical construction of variables by the Markowitz model, as well as the establishment of investment scheme and return prediction under different circumstances. It clearly tests that the Markowitz model has the practicability of investment suggestions for contemporary investors.

1. Introduction

In the 1960s, based on the development of the modern portfolio theory that is developed by William, John, Jack, and Mossin, CAMP (Capital Asset Pricing Model) was born [1], the asset pricing model that advocates that the rate of return of the portfolio is only related to systemic risk and that the model portfolio must be polymerized, that is, including all risky assets to eliminate non-systemic risks of individual securities. Based on the development of camp, Black, Jensen, and Scholes studied the shortcomings of CAPM when risk-free assets do not exist [2], and proposed zero β Model, that is, when risk-free assets do not exist, COV (RZ, RM) is used as a substitute for risk-free assets. After that, according to the continuous process of the securities market, Morton, Heath and Jarrow occasionally built a theoretical framework of continuous-time portfolio and asset pricing [3], which gave birth to the inter-temporal capital asset pricing model (ICAPM). Until 1978, Lucas and Breeden put forward the consumption-based asset pricing model (CCAPM) [4]. By describing the risk with the covariance between asset return and consumption growth rate, combining consumption choice theory and asset pricing theory, it has become the basic theory to study consumers' intertemporal choice behavior. After that, American economists Ross put forward the arbitrage pricing theory [5], assuming that in the perfect competition and frictionless capital market, the single period expectation rate of any risky asset is related to its related risk factors.

Since entering the 21st century, research on asset pricing models has not stopped, Dionne and Okou showed that for a risk-averse representative agent, it is the first-degree expectation dependence (FED) rather than the covariance that determines C-CAPM's riskiness [6]. Of course, the CAPM model still has some defects, so Fard and Falah proposed to analyze the situation after changing the company's strategy by using two new beta values, long-term beta values, as well as short-term beta values [7]. Based on the development of China's securities market, Zhang and Meng started the empirical research of CAPM in the Chinese market [8]. However, they found that the CAPM model does not apply to the Chinese stock market [9], because the Chinese stock market is still in the development process, which is not mature, and there is a certain gap from the market standards.

By changing the weights of different stocks in Markowitz model, aiming at different investment purposes and the constraints existing in combination with the actual situation, such as the maximum sharp ratio and the standard deviation of the minimum return on investment, we can get different investment weights, test whether it meets the investment purpose and obtain the ideal investment result, And provide other investors with examples that match the actual situation and provide them with suggestions for pursuing the best return on investment.

The remainder of the paper is organized as follows: Section 2 summarizes and analyses the data used in the past 20 years. Additionally, it briefly analyses the stock price trend of the selected

companies, and analyses the reasons for choosing the food industry, technology industry and traditional energy industry; Section 3 is the construction method and data of Markowitz model; Section 4 is that according to the possible needs of various investors in the future, five constraints in line with the actual situation are set; The last section presents our conclusions.

2. DATA

2.1. Data introduction

This article mainly uses 20 years of daily data of total returns for ten stocks and S&P 500 index for analysis. And the reason for selecting SPX is that it contains more companies, and therefore the risks are more fragmented and can reflect broader market changes. The S&P 500 index covering all of the companies, is a company listed on the main exchange in the United States. And because it contains the company a lot, so it the index values of very precise, also has very good continuity, can reflect the very wide range of market changes, and therefore is widely considered to be an ideal target of stock index futures contracts, also became one of the benchmark index of the portfolio.

QUALCOMM Incorporated engages in the development and commercialization of foundational technologies for the wireless industry worldwide and it was incorporated in 1985. Akamai Technologies, Inc. provides cloud services for securing, delivering, and optimizing content and business applications over the internet in the United States and internationally, and it was founded in 1998. Oracle Corporation provides products and services that address enterprise information technology environments worldwide, which was founded in 1977. Last, Microsoft Corporation develops, licenses, and supports software, services, devices, and solutions worldwide. Microsoft Corporation was founded in 1975 and is based in Redmond, Washington.

The above is information technology industry, this is in the last 40 to 50 years of emerging industries, has played an extremely important role. Additionally, Robinson, Schulz, Dunn, Casilli, Tubaro, Carveth and Khilnani have reported that according to the changes in social demand in recent years, as information utilization, data mining services, the Internet and the development of the information age brings huge opportunities for these enterprises [10], portfolio of information technology industry companies is a good choice, but the industry has greater industry volatility, that is, higher returns with higher risk.

Additionally, Chevron Corporation, founded in 1879, through its subsidiaries, engages in integrated energy, chemicals, and petroleum operations worldwide. And Exxon Mobil Corporation explores for and produces crude oil and natural gas in the United States and internationally. As of December 31, 2020, it had approximately 22,239 net operated wells with proved reserves, and the company was founded in 1870. For the Imperial Oil Limited, it mainly explores for, produces, and sells crude oil and natural gas in Canada. It operates through three segments: Upstream, Downstream, and Chemical. And The Company was incorporation in 1880.

For these energy industries, they are more of the traditional energy industry and for the energy industry such as natural gas, Hewitt, Bradley, Baggio Barlagne, Ceglarz, Cremades and Slee found that in European, although the society has advocated the development of new energy, but the status quo is the traditional energy market proportion still occupies a very large share, so for the energy industry investment, more attention in the traditional energy industry is reasonable, and they have great foundation to develop new energy industry, so we think the industry has less risk and returns

The Coca-Cola Company, founded in 1886, a beverage company, manufactures, markets, and sells various nonalcoholic beverages worldwide, and it operates through a network of independent bottling partners, distributors, wholesalers, and retailers, as well as through bottling and distribution operators. For PepsiCo, Inc., it operates as a food and beverage company worldwide and operates through seven segments and distributor networks, as well as directly to consumers through e-commerce platforms and retailers. And PepsiCo was and is headquartered in Purchase, New York. Last, McDonald's Corporation, founded in 1940, operates and franchises McDonald's restaurants in the United States

and internationally. Its restaurants offer various food products and beverages, as well as breakfast menu. As of December 31, 2020, the company operated 39,198 restaurants.

Finally, traditional food and beverage manufacturing industry, especially Coca-Cola and Pepsi, is the world retail food leader, the development of these enterprises is very stable, and to meet the needs of people's constantly, which is the growing population provides the basis for the value growth of these companies. So, the risk of the industry's returns is small

2.2. Analysis of stock price change

Figure 1. SPX Price

SPX is the stock index of 500 medium and large listed companies in the U.S. stock market. By dividing the average price of stocks in the reporting period and the average price of stocks in the selected base period, the ratio obtained is multiplied by the index value of the base period, which becomes the U.S. standard & Poor's index. Because it contains many companies, its index value is very accurate, it also has good continuity and can reflect a wide range of market changes. Carta, Corriga, Ferreira, Podda and Recupero have concluded that S&P is generally considered to be the subject of an ideal stock index futures contract. At the same time, it has also become the benchmark of the U.S. portfolio index. On the whole, the overall trend of the growth of U.S. enterprises is continuous and rising steadily.
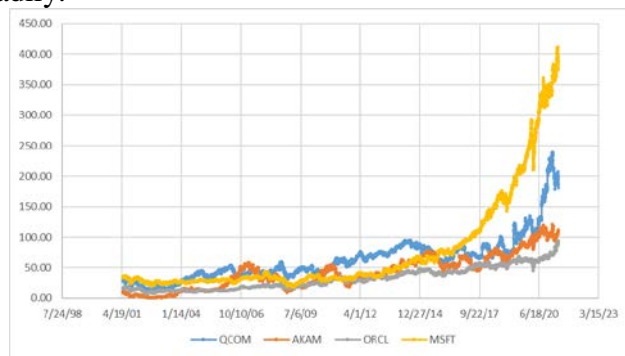

Figure 2. Technology Industry Stock Price

The above four curves are mainly companies in the technology industry. In the past 20 years, it is mainly the development peak of industries such as data analysis, mechanical software and chip technology. Therefore, Microsoft has become the company with the best growth performance in the past 20 years. At the same time, other companies also show very good investment, and after many turbulences, in particular, Akamai and JPMorgan Chase have still completed their transformation through many technological changes. Finally, Oracle will still show extremely strong growth after completing the transformation of traditional industries
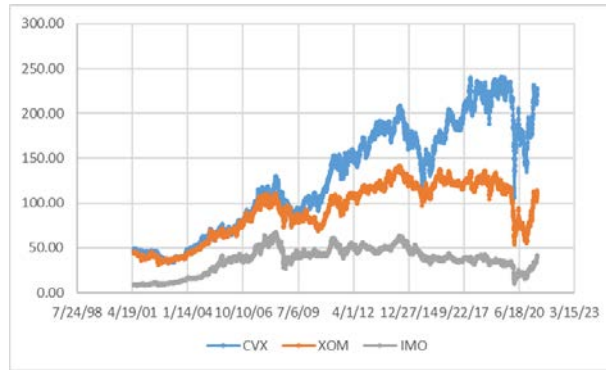
Figure 3. Energy Industry Stock

As a traditional energy industry company, it is one of the basic industries of social development. However, with the development of new energy, the traditional energy industry began to face severe challenges after entering 2017. Almost the three companies threw away the impact of market value, and the Centennial trend of stock price showed amazing similarity. It is known that in 2020, the three companies began to enter the new energy industry almost at the same time, I believe that in the future, because the international energy industry is changing from traditional energy to clean energy, and affected by social demand, the value of the company has ushered in a turnaround.

Figure 4. Food Industry Stock Price

Coca Cola, Pepsi and McDonald's are old brand food industries in the United States. Due to the increase of population, the increase of people's demand, the development of service industry, products and branch products and the continuous change in line with public consumption habits, these three food industries have shown a very stable development trend in the past 20 years.

Table 1. Correlations Table

Correlations	SPX	QCOM	AKAM	ORCL	MSFT	CVX	XOM	IMO	KO	PEP	MCD
SPX	1.000	0.557	0.389	0.546	0.639	0.613	0.568	0.522	0.491	0.522	0.537
QCOM	0.557	1.000	0.278	0.285	0.375	0.233	0.235	0.272	0.197	0.263	0.262
AKAM	0.389	0.278	1.000	0.242	0.256	0.122	0.068	0.127	0.085	0.102	0.291
ORCL	0.546	0.285	0.242	1.000	0.475	0.264	0.301	0.233	0.068	0.205	0.137
MSFT	0.639	0.375	0.256	0.475	1.000	0.339	0.304	0.250	0.279	0.334	0.357
CVX	0.613	0.233	0.122	0.264	0.339	1.000	0.829	0.734	0.402	0.272	0.394
XOM	0.568	0.235	0.068	0.301	0.304	0.829	1.000	0.697	0.338	0.240	0.340
IMO	0.522	0.272	0.127	0.233	0.250	0.734	0.697	1.000	0.297	0.178	0.268
KO	0.491	0.197	0.085	0.068	0.279	0.402	0.338	0.297	1.000	0.579	0.499
PEP	0.522	0.263	0.102	0.205	0.334	0.272	0.240	0.178	0.579	1.000	0.470
MCD	0.537	0.262	0.291	0.137	0.357	0.394	0.340	0.268	0.499	0.470	1.000

We can see that in the correlation table, the correlation between each stock and SPX is about 0.5. Compared with the correlation between stocks in other different industries, there is a strong correlation between different stocks and SPX.

By comparing different stocks in the same industry, we can see that the relationship coefficient between stocks, MCD, PEP and Ko in the food industry is about 0.5. It can be seen that there is a strong correlation between stocks in the beverage and food industry. The same correlation is also shown among CVX, XOM and IMO, but compared with different stocks in the technology industry, The average value of correlation is only about 0.3.

By comparing the stock relationship between different industries, it can be found that, relatively speaking, the stock correlation between food and traditional energy industry is stronger than that between food industry and technology industry, as well as traditional energy industry and technology industry.

3. Method

The Markowitz model was used to explore the optimal portfolio of different stock portfolios under different constraints, and the results were compared. It is beneficial for investors to make decision analysis.

3.1. Five Constraints

$$\sum_{i=1}^{7} |w_i| \leq 2 \tag{1}$$

For a given total capital, the leverage multiple of making an investment is not allowed to be more than twice, and in this case allows the number of leverages for a single investment to be too large or short

$$|w_i| \leq 1 \; for \; all \; i \tag{2}$$

For all stocks invested, any more than double leverage is not allowed for a single stock when short selling is allowed

$$\text{None (free)} \tag{3}$$

All stocks are free to be sold and brought

$$w_i \geq 0 \; for \; all \; i \tag{4}$$

In some cases, short trading of any stock invested is not allowed, but leveraged trading of all stocks is allowed

$$w_1 = 0 \tag{5}$$

Under this limitation, no transaction is made on the first stock invested, and in actual cases we may not trade on any stock in a given case

3.2. Markowitz Model

Expected yield is the expected value of the possible future yield, also known as the expected yield. The future state of the single securities is uncertain, and the return in each state is also different, with the expected yield to represent the expected return. Similarly, the return of multiple securities is represented in the same way.

3.2.1. Expected earnings for single securities

The expected return of a single securities, such securities in a state in the future, then the expected return of the securities is:

$$E(r_i) = \sum_{s=1}^{N} r_{is} p_s, \; \sum_{s=1}^{N} P_s = 1 \tag{6}$$

$E(r_i)$ the expected return;
p_s is the probability of the S appearing;
r_{is} is the return of the securities i when the situation S arises;
N is the total number of various possible conditions;

3.2.2. Expected earnings for the securities portfolio

The expected return of the securities portfolio can be obtained after the expected return of a single securities, r_p Indicates the weighted average of the expected returns of the various assets included in the portfolio, as expressed as:

$$E(r_p) = \sum_{i=1}^{N} x_i E(r_i), \sum_{i=1}^{N} x_i = 1 \tag{7}$$

$E(r_p)$ the expected yield of the securities portfolio
$E(r_i)$ is the expected return of the securities i in the portfolio;
x_i is proportion of the securities i in the portfolio
N is type of securities in the portfolio

3.2.3. Risk about Markowitz Model

In Markowitz theory, risk is defined as the volatility of investment returns. The more volatile the yield is, the higher the risk of making an investment. Volatility of the yield, usually expressed by the standard deviation or variance.
(1) The expected risk of a single securities i
The variance and standard deviation, is calculated as follows:

$$\text{Variance}: \; \sigma_i^2 = \sum_{s=1}^{N}[r_{is} - E(r_i)]^2 p_s \tag{8}$$

Standard deviation:

$$\sigma_i = \sqrt{\sum_{s=1}^{N}[r_{is} - E(r_i)]^2 p_s} \tag{9}$$

(2). the expected risk of the securities portfolio

Covariance

The risk of the securities portfolio is not only related to the risk of each security, but the mutual relationship between the securities also has an impact on the risk of the portfolio. The uncertainty in the gains resulting from mutual influence between securities can be expressed by covariance. Covariance is a statistic for measuring the interactivity between two random variables such as the yield of securities i and the yield of securities j.

σ_{ij} Represents the covariance between the securities i and j, then:

$$\sigma_{ij} = \sigma_{ji} = E[(r_{is} - E(r_i))(r_{js} - E(r_j))] \tag{10}$$

If the covariance between the two securities is positive, indicating that the yields of both securities tend to change in the same direction, namely that the actual yield of one of the securities is higher than the expected yield may occur accompanied by the same situation of the other securities. If the covariance between the two securities is negative, there is a reverse variable relationship between the two securities, and the rising yield of one security may be accompanied by a decline in the yield of the other. A relatively small or zero covariance indicates that only a small interaction between two securities or no interaction is independent of each other. The greater the covariance between the securities, the greater the risk of the portfolio of securities constituted by them.

Correlation Coefficients

The return interaction between the two securities can also be represented by another statistic, namely the correlation coefficient between the two.

Assume σ_i and σ_j is the return standard deviation of securities i and j respectively, σ_{ij} is the covariance between the two securities, and then the calculation formula of ρ_{ij} is:

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j} \tag{11}$$

The range of ρ_{ij} is $-1 \leq \rho_{ij} \leq 1, \rho_{ij} = -1$ It indicates that the change direction of the two securities income results is completely different, which is called completely negative correlation; $\rho_{ij} = 1$ It indicates that the change direction of the two securities return results is exactly the same, which is called full positive correlation; $\rho_{ij} = 0$ There is no relationship between the changes in the earnings outcome of the two securities; when ρ_{ij} in the interval of $(-1,0)$, In indicates that the results of the two securities earnings change in the opposite direction, but not 100 percent completely opposite, only a general negative correlation. when ρ_{ij} in the interval of $(0,1)$ In means that the results of both securities earnings change in the same direction, but not 100 percent exactly the same, only a general positive correlation. And when the ρ_{ij} is equal 0, That is, securities i and securities j are unrelated only show that there is no linear correlation between securities i and securities j, but do not exclude other forms of (non-linear) dependence of securities i and securities j.

Generally speaking, if the correlation coefficient between the two securities occurs $\rho_{ij} < 0$, it may reduce the investment risk after the portfolio, and if the correlation risk coefficient occurs between them $\rho_{ij} > 0$, it may increase the investment risk after the portfolio.

Variance and standard deviation of the securities portfolio

The expected risk of the portfolio σ_p^2 is :

$$\sigma_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \sigma_{ij} \tag{12}$$

The standard deviation σ_p:

$$\sigma_p = \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \sigma_{ij}} \tag{13}$$

And when i is not equal j, σ_{ij} It indicates the covariance of the returns of securities i and j, reflecting the correlation of the returns of both securities change in a common cycle, xi and xj indicate the

proportion of securities i and j in the portfolio；when i is equal j, $\sigma_{ij} = \sigma_i{}^2$ represents the variance of the securities i earnings

4. Result analysis
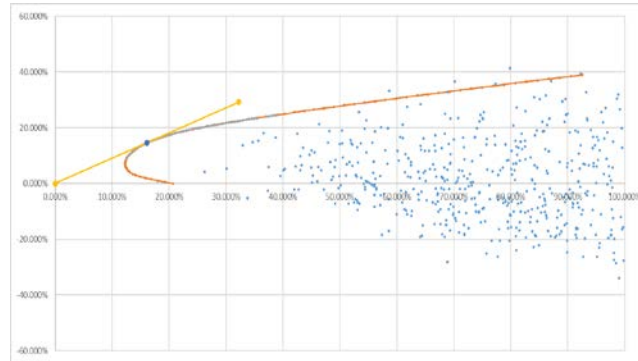
4.1. First Constraint

Figure 5. First Constraint Analysis

Figure 5 is under the first constraint; the leverage multiple of investment is not allowed more than twice for a given investment. The cut between the CAL and the highest yield and the lowest risk portfolio we pursue, which we can clearly see is the blue point in the figure that also shows that the Sharp ratio reached the maximum at this time of 0.905, about 14.6% and a standard deviation of about 0.16

Table 2. Weight Table of Constraint 1

weight	SPX	QCOM	AKAM	ORCL	MSFT	CVX	XOM	IMO	KO	PEP	MCD
MinVarance	0.310	-0.022	-0.010	0.053	-0.004	-0.087	0.196	-0.039	0.210	0.305	0.089
MaxSharp	-0.409	0.060	0.062	0.149	0.209	0.058	-0.091	0.115	0.051	0.290	0.506

Table 3. Investment information of constraint 1

	Return	StDev	Sharpe
MinVarance	7.232%	12.279%	0.589
MaxSharp	14.587%	16.112%	0.905

We get different weights according to the investment return; we will get different weights. When investors pursue the minimum investment change, 0.022 of total short selling QCOM investment, 0.0105 of total short selling AKAM investment, 0.0038 of total short selling MSFT investment, 0.087 of total short selling CVX investment and the proportion of investment in IMO is 0.039. The same investment in SPX, ORCL, XOM, KO, PEP and MCD respectively, expecting an investment return of 7.232.

Similarly, when investors pursue the maximum investment Sharp ratio, they will choose to short sell SPX and XOM, and invest in other companies, according to the obtained investment weight, so as to get the optimal investment results obtained between CAL and the effective frontier
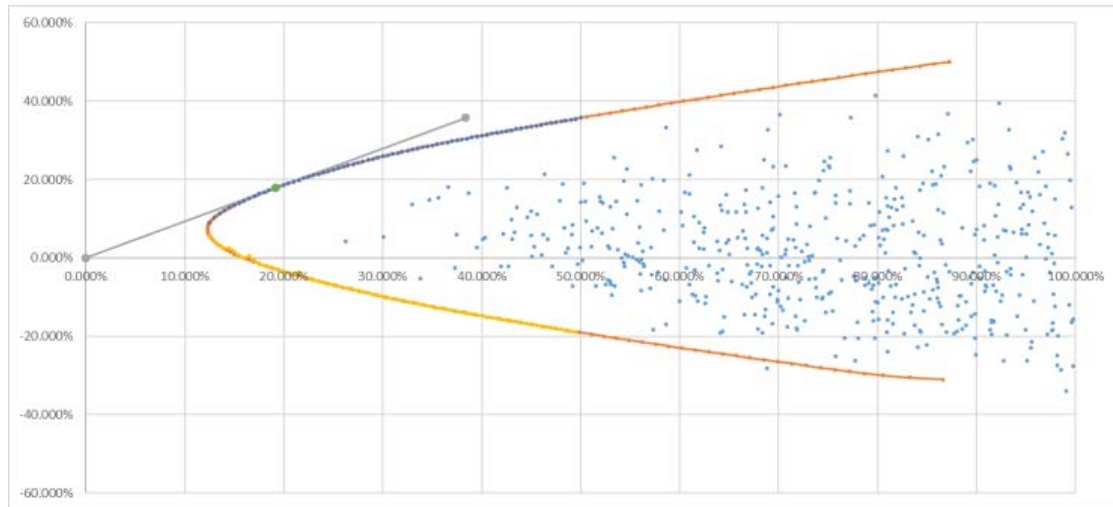
4.2. Second Constraint

Figure 6. Second Constraint Analysis

In short selling and buy a single stock investment leverage does not allow more than double, we get CAL and effective boundary (blue line), the two will be cut to the green point, to the maximum return on investment, the minimum investment risk, investment return is about 17.9%, investment standard deviation (investment change) is about 19.189%, the maximum sharp ratio is 0.933

Table 4. Weight Table of Constraint 2

weight	SPX	QCOM	AKAM	ORCL	MSFT	CVX	XOM	IMO	KO	PEP	MCD
MinVarance	0.310	-0.022	-0.010	0.053	-0.004	-0.087	0.196	-0.039	0.210	0.305	0.089
MaxSharp	-1.000	0.122	0.084	0.242	0.304	0.261	-0.300	0.181	0.095	0.385	0.626

Table 5. Investment information of constraint 2

	Return	StDev	Sharpe
MinVarance	7.232%	12.279%	0.589
MaxSharp	17.900%	19.189%	0.933

We get different weights according to the investment returns; we will get different weights. When investors pursue the minimum investment change, 0.022 of total short selling QCOM investment, 0.0105 of total short selling AKAM investment, 0.0038 of total short selling MSFT investment, 0.087 of total short selling CVX investment and the proportion of investment in IMO is 0.039. The same investment in SPX, ORCL, XOM, KO, PEP and MCD respectively, expecting an investment return of 7.232.

Similarly, when investors pursue the maximum investment Sharp ratio, they will choose to short sell SPX and XOM, and invest in other companies according to the investment weight, so as to get the optimal investment results between CAL and the effective boundary. The return is 17.9%, and the standard deviation of investment return is 19.189%

4.3. Third Constraint

We can see that the best investment result for investment in the free investment state is 18.147%, and the Sharp ratio is 0.933. Similarly, the standard deviation of investment return is 19.451, which is

the cut point of the CAL curve and the effective boundary in Figure C, Table 6. Weight Table of Constraint 3

Which represents the most investment plan. Then the corresponding investment plan is SPX and XOM for short selling, and the SPX has short leverage and effective investment in other companies, then we get our investment plan.

If investors only pursue the minimum change in return on investment under this restriction, we can get a return of 7.232%. At the same time, the investment weight of each stock is mainly short against QCOM, AKAM, MSFT, CVX and IMO, but also for the allocation of a given investment in other stocks
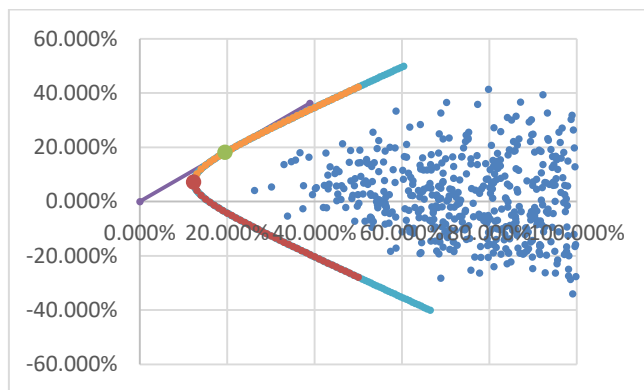

Figure 7. Third Constraint Analysis

Weight	SPX	QCOM	AKAM	ORCL	MSFT	CVX	XOM	IMO	KO	PEP	MCD
MinVarance	0.310	-0.022	-0.010	0.053	-0.004	-0.087	0.196	-0.039	0.210	0.305	0.089
MaxSharp	-1.052	0.128	0.086	0.249	0.313	0.270	-0.305	0.185	0.097	0.392	0.637

Table 7. Investment information of constraint 3

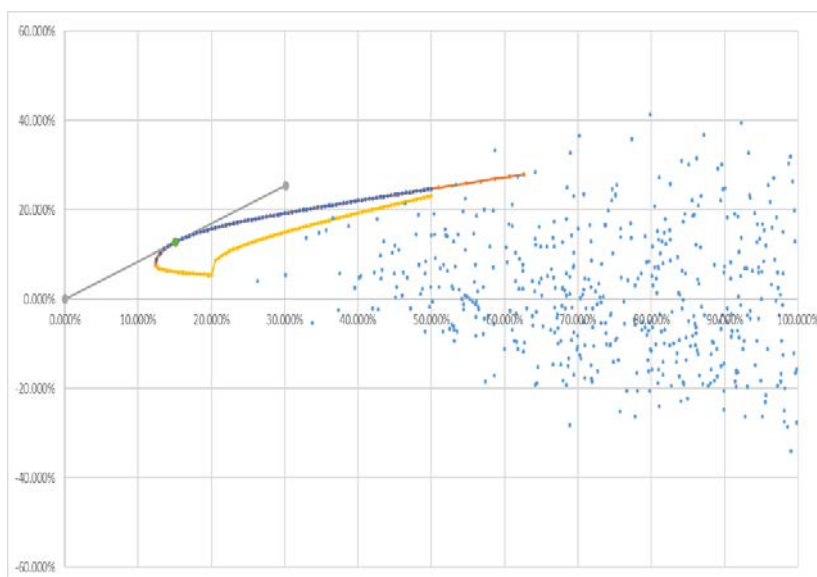|	Return	StDev	Sharpe
MinVarance	7.232%	12.279%	0.589
MaxSharp	18.147%	19.451%	0.933

4.4. Fourth Constraint

Figure 7. Fourth Constraint Analysis

Under the fourth constraint, the investment is not allowed to short, but allowed not to operate on the stock, while the leverage multiple is not set for a single stock, we can get the gray CAL line and effective boundary (purple line) of about 12.759%, and the standard variance of 15.065%, the maximum Sharp ratio of 0.847

Table 8. Weight Table of Constraint 4

weight	SPX	QCOM	AKAM	ORCL	MSFT	CVX	XOM	IMO	KO	PEP	MCD
MinVarance	0.197	0.000	0.000	0.063	0.000	0.000	0.112	0.000	0.210	0.335	0.083
MaxSharp	0.000	0.021	0.053	0.084	0.148	0.000	0.000	0.053	0.000	0.197	0.443

Table 9. Investment information of constraint 3

	Return	StDev	Sharpe
MinVarance	8.025%	12.398%	0.647
MaxSharp	12.759%	15.065%	0.847

Depending on the needs of investors, when investors are not allowed, if the standard deviation of return on investment, then 12.398%, in this case only for SPX, ORCL, X O L, XOM, KO, PEP and MCD to continue to invest, and finally achieve a return on investment of 8.025%

When investors expect to achieve maximum return on investment, minimum investment risk and maximum Sharp ratio, the investment recommendation is to invest in QCOM, AKAM, ORCL, MSFT, IMO, PEP and MCD, and the rest of the stock is not operated to achieve the best portfolio.
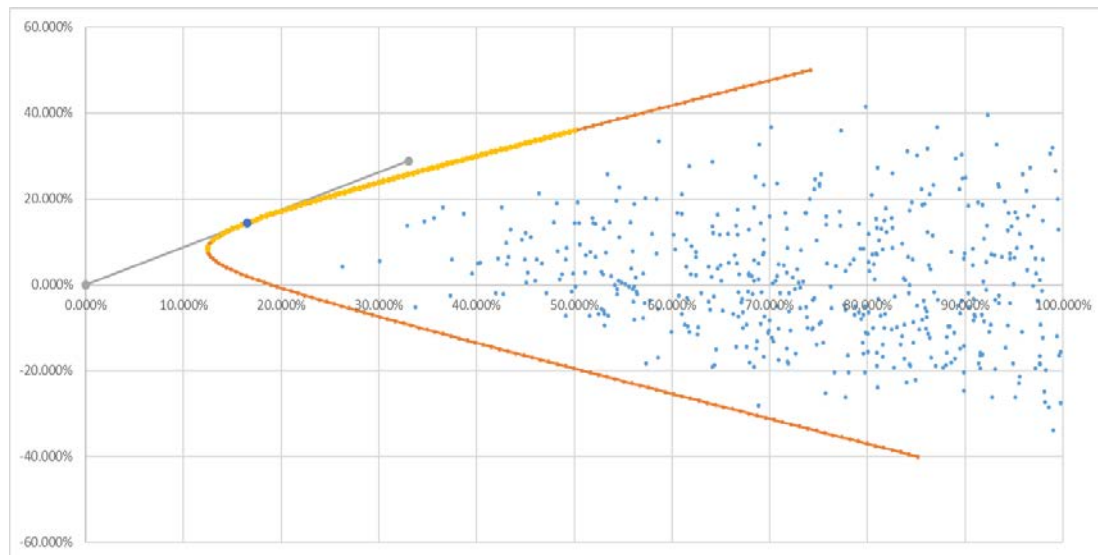
4.4. Fifth Constraint

Figure 8. Fifth Constraint Analysis

Set only a single stock to not have any trade, the example provided is no trading operation on SPX only, but there are no restrictions on the investment in the rest of the stock, allowing short selling and leverage, in practice, we can assume no trading on any stock.

Table 10. Weight Table of Constraint 5

weight	SPX	QCOM	AKAM	ORCL	MSFT	CVX	XOM	IMO	KO	PEP	MCD
MinVarance	0.000	0.008	-0.001	0.092	0.037	-0.051	0.218	-0.028	0.248	0.361	0.116
MaxRatio	0.000	0.024	0.048	0.116	0.162	0.130	-0.327	0.133	-0.009	0.217	0.506

Table 9. Investment information of constraint 3

	Return	StDev	Sharpe
MinVarance	8.162%	12.457%	0.655
MaxRatio	14.383%	16.537%	0.870

According to the Table, what we can see is that the cut point between CAL and the effective boundary is 14.383%, and the return-on-investment standard deviation is 16.537%, when the weight of investment is allocated to the short selling of XOM and KO, the weight of several other stocks is determined by the return on investment, and then realized the maximum return on investment and the minimum investment risk, and reached the maximum Sharp ratio of 0.87

According to investor needs, if the minimum standard deviation for pursuing smaller return on investment is floating, short AKAM, CVX, AM and IMO, while making any transactions on SPX.

5. Conclusion

Based on the continuous development of the modern investment theory, People began to constantly revise the CAPM, And develop the market application of CCAPM and CAPM according to different situations, But we find that the basis of the modern investment theory is still the Markowitz model, We therefore used the most basic Markowitz model to test the portfolio, Explore whether it still applies to the modern investment environment, And whether it can meet certain investment needs, So as to provide a reference for modern investors, By assuming the demand of the investors, Assuming to pursue a maximum Sharp rate, Or to pursue minimal investment changes, Solve the investment scheme problem under certain restrictions, This will provide a reference for other investors.

Finally, the creation of the Markowitz model was hailed as a scientific revolution in financial theory. But any excellent theory has its limitations, and we need to take a comprehensive view of the Markowitz model. The first is that the research method uses the expected benefits and the expected risks to consider the actual benefits and risks, And covariance estimates from previous data to measure the correlation between securities, Because historical digital data does not accurately reflect the situation of future benefits and risks, Various variables of a securities also vary over time, So there may still be some uncertainty about the future prediction; in addition, Due to the different digital analysis, as well as the actual situation, Even with the gains and risks of the various securities at some point, Due to the wide variety of securities, Too much computation leads to give optimal investment advice; In terms of mathematical computing, The model measures the magnitude of the risk using variance or standard deviation in expected yield changes in the future. So although the size of the risk is clear and easy to measure, but due to the variance and standard deviation in calculation, will be expected yield benefit investors into the category of risk, this is questionable, of course, with variance as a measure of asset risk this is only applicable to symmetrical distribution of asset income, does not have general. Therefore, in practice, we will still explore the use of half-variance for portfolio analysis, and finally the Markowitz optimal portfolio is only in a temporary portfolio, namely requires all investors to have a common single investment period, all the securities portfolio has a unique holding period, but this is not done in real life. The financial environment is changing rapidly, and the portfolio derived by the model may not be the optimal solution within the specified period. Therefore, in future studies, we still need to explore how to solve the differences in the data in the historical background, the data specificity problems, and the impact of the investment cycle on the investment results.

References

[1] Liu, J. H. (2013). Reviews and a new angle of β coefficient prediction method. In *Advanced* Materials Research (Vol. 709, pp. 687-690). Trans Tech Publications Ltd.

[2] Black, F., Jensen, M. C., & Scholes, M. (1972). The capital asset pricing model: Some empirical tests.

[3] Heath, D., Jarrow, R., & Morton, A. (1992). Bond pricing and the term structure of interest rates: A new methodology for contingent claims valuation. *Econometrica: Journal of the Econometric Society*, 77-105.

[4] Roberti, C. F. (2019). An implementation of the consumption-based capital asset pricing model.

[5] Ross, S. A. (1978). A simple approach to the valuation of risky streams. *Journal of business*, 453-475.

[6] Dionne, G., Li, J., & Okou, C. (2012). An extension of the consumption-based CAPM model. *Available at SSRN 2018476*.

[7] Fard, H. V., & Falah, A. B. (2015). A New Modified CAPM Model: The Two Beta CAPM. *Jurnal UMP Social Sciences and Technology Management*, *3*(1).

[8] Zhang, P., & Meng, X. H. (2013). The market application analysis of CAPM model. In *Applied* Mechanics *and Materials* (Vol. 380, pp. 4422-4425). Trans Tech Publications Ltd.

[9] Zhang, P., & Meng, X. (2013, April). The market application analysis of CAPM model in China's securities. In *2nd International Conference System Engineering and Modeling (ICSEM-13). Atlantis Press*.

[10] Robinson, L., Schulz, J., Dunn, H., Casilli, A., Tubaro, P., Carveth, R., ... & Khilnani, A. (2020). Digital inequalities 3.0: Emergent inequalities in the information age. *First Monday*, *25*.

[11]

[12] Hewitt, R. J., Bradley, N., Baggio Compagnucci, A., Barlagne, C., Ceglarz, A., Cremades, R., ... & Slee, B. (2019). Social innovation in community energy in Europe: A review of the evidence. *Frontiers in Energy Research*, *7*, 31.

[13] Carta, S., Corriga, A., Ferreira, A., Podda, A. S., & Recupero, D. R. (2021). A multi-layer and multi-ensemble stock trader using deep learning and deep reinforcement learning. *Applied Intelligence*, *51*(2), 889-90